A short course on Generalized Hydrodynamics

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It is a hard problem to describe or solve physical systems composed of very many particles in interactions, or other degrees of freedom such as spins or fields. In fact, there is no point obtaining each particle's trajectory, or the system's precise wave function, as these are in most situations essentially un-observable. Only certain "aggregate" quantities are relevant at large scales of space and time. What are the emergent degrees of freedom? Is there an associated emergent dynamics that describes them, without the need for the underlying fundamental equations from classical or quantum mechanics? Taking a perhaps un-conventional viewpoint, one can see the theory of hydrodynamics as a universal framework for such emergent dynamical behaviours in many-body systems.

In these lectures, I will overview the general ideas behind hydrodynamics and apply them to one-dimensional systems that have the property of integrability. Integrable systems possess an extensive number of conservation laws, and have other special structures that make them amenable to exact calculations. More than being toy models of physics, they are experimentally realisable for instance in cold atomic gases. Most importantly, as we have discovered recently, they fall into a new universality class of emergent large-scale dynamics, where the standard Euler and Navier-Stokes equations fail. This universality class is described by a theory which we refer to as Generalised Hydrodynamics (GHD). I will explain how combining the conservation laws with the scattering properties of integrable systems lead to the main equation of GHD, and, if time permits, discuss how to account for diffusive effects, evaluate correlations, and describe fluctuations. I will perhaps even give ideas for how to go to higher dimensions. For illustrations, I will concentrate on simple models: the classical hard rods (and some generalisations), and the experimentally relevant quantum Lieb-Liniger model.

An approximate plan is:

Lecture 1: Relaxation, thermodynamics and hydrodynamics in many-body systems
Lecture 2: Integrable systems: conservation laws and the scattering picture
Lecture 3: Euler hydrodynamics of integrable systems
Lecture 4: Going further: diffusion, correlations, fluctuations, higher dimensions?

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